Complex Geometry at Large

August 12 – 16, Sunny Beach, Bulgaria

The conference is funded by Simons Foundation Grants of ICMS-Sofia and IMSA-Miami and the Scientific Programme PIKOM, Ministry of Education and Science of the Republic of Bulgaria.

Rodolfo Aguilar *Infinitesimal methods in mixed Hodge theory*

After recalling and giving examples of infinitesimal variations of Hodge structures, we define its mixed counterpart. Then we define some invariants and describe them more explicitly in the case of pairs (X, Y) where X is a Fano 3-fold and Y an anticanonical surface. We will conclude with a Torelli type theorem for general pairs (X, Y) where X is a cubic threefold and Y as before. Joint work with M.Green and Ph. Griffiths.

Karim Adiprasito *On a conjecture of Oda and Alexander*

Given two birational toric varieties, we show that there is a common weighted blowup to both of them. This resolves conjectures of Alexander and Oda, respectively. Joint with Igor Pak.

Dennis Borisov *spin(7) and su(4) connections on Calabi-Yau 4-folds*

Given a compact Calabi-Yau 4-fold and a Hermitian vector bundle E on it, one can construct two moduli spaces of connections: those with curvature in su(4) (i.e. holomorphic structures) and those with curvature in spin(7). I will discuss a relationship between these moduli spaces, that involves shifted symplectic structure and Lagrangian foliation. This is a gauge-theoretic construction that should correspond to a choice of Lagrangian foliation on the moduli space of coherent sheaves on the algebraic geometric side.

Morgan Brown *The complement of the Essential Skeleton of a Berkovich Space*

Berkovich spaces give a formalism for constructing spaces of valuations on varieties over nonarchimedean fields. As such they encode a great deal of information from birational geometry. The most notable invariant is the essential skeleton, a subset of the Berkovich space corresponding to the valuations monomial on strata of a dlt minimal model of X. Inspired by Mori's conjecture in birational geometry, we conjecture that the essential skeleton is the complement of the images of all open fiber disks, which are analytic objects analogous to families of rational curves. This is joint work with Jiachang Xu and Muyuan Zhang.

Ugo Bruzzo *Stable Supermaps*

I will review the construction and some properties of the stack of stable supermaps.

Leonardo F. Cavenaghi *Atoms of Stacks*

In this talk, we quickly recall the concept of atoms from Katzarkov–Kontsevich–Pantev–Yu. This Gromov–Witten-based construction recently led to new birational invariants. We explain how this idea can be generalized to produce birational invariants for stacks and G-birational invariants for smooth projective varieties with regular G-actions. This talk is based on ongoing joint work with L. Grama, L. Katzarkov, and M. Kontsevich.

Peter Dalakov *The G_2 Donagi–Markman cubic and Seiberg–Witten differentials on the Hitchin base*

In this talk I will outline the derivation, together with U.Bruzzo, of a formula for the Donagi–Markman cubic for the G_2-Hitchin system. I will also present another proof of the result, originally due to Hitchin, that the cubic is invariant under the Langlands involution of the base, using a recent work of ours.

Oscar Garcia-Prada *Non-maximal Toledo components*

The well-known Milnor–Wood inequality gives a bound on the Toledo invariant of a representation of the fundamental group of a compact surface in a group of Hermitian type. While a lot is known regarding the counting of maximal Toledo connected components, and their role in higher Teichmueller theory, the non-maximal case remains elusive. In this talk, I will outline a strategy to count the number of such non-maximal Toledo components. This is joint work in progress with Brian Collier and Jochen Heinloth.

Lino Grama *Spherical T-duality meets exotic Spheres and Homotopy Hopf Manifolds*

The concept of Spherical T-duality aiming to generalize the classical well-studied T-duality. Shortly saying, for spherical T-duality, the role of the circle S^1, or the circle group $\mathrm{U}(1)$, is replaced by the 3-sphere S^3, or the special unitary group $\mathrm{SU}(2)$. On the one hand, topological T-duality relates pairs consisting of total spaces of $\mathrm{U}(1)$-principal bundles equipped with a cocycle in degree-3 ordinary cohomology; on the other hand, spherical T-duality relates pairs consisting of $\mathrm{SU}(2)$-principal bundles equipped with cocycles in degree-7 cohomology.

In this talk, we compare the two concepts and explain how 7-dimensional homotopy spheres are inserted in the context of spherical T-duality. Particularly, we show that a class of examples of the manifolds in a spherical

T-duality diagram consists in $\Sigma^7 \times S^1$, where Σ^7 is a homotopy sphere. Furthermore, any complex structure on $\Sigma^7 \times S^1$ with a chosen holomorphic structure can be transported to a different choice of diffeomorphism class of $\Sigma^7 \times S^1$ – such a phenomenon does not occur for $S^3 \times S^1$, that is, this Hopf manifold does not admit two distinct holomorphic structure under T-duality. The results come from a joint work with Leonardo Cavenaghi and Ludmil Katzarkov.

Gueo Grantcharov *Non-Kähler manifolds via mapping tori*

I'll report on some results about the existence of special Hermitian structures, like balanced, CYT, and SKT on mapping tori (also called toric suspensions). I'll start with a construction of a toric suspension of hyperkaehler manifold, obtained in collaboration with Anna Fino and Misha Verbitsky. Next, I will discuss joint results with Beatrice Brienza and Anna Fino on similar construction of compact complex manifolds admitting a Hermitian structure which is SKT and CYT, but not Bismut flat.

Phillip Griffiths *Atypical Hodge Loci*

In recent years there has been a flurry of developments in the subject of functional trancendence. Novel techniques from model theory (o-minimality) have led to results including the Ax–Schanuel conjecture for period mappings and atypicality of Hodge loci for non-classical period images. This talk will discuss the proof of the atypicality result and its relation to Ax–Schanuel for period mappings.

Richard Paul Horja TBA

Jochen Heinloth *Geometry of G-Hodge bundles on curves*

In joint work with Brian Collier and Oscar Garcia-Prada we found an algebraic interpretation of the particular stability conditions for G-Hodge bundles introduced by Biquard, Collier, Garcia- Prada and Toledo in terms of theta-stability. After recalling this description I will try to explain how we can use this interpretation to understand the geometry of the corresponding moduli spaces through wall-crossing.

Stefan Ivanov *Almost Calabi–Yau with torsion 6-manifold, curvature and generalized Ricci solitons*

It is observed that on a compact almost complex Calabi–Yau with torsion 6-manifold (ACYT) the Nijenhuis tensor is parallel with respect to the torsion connection. If the torsion is closed then the space is a compact generalized gradient Ricci soliton and this is equivalent to a certain vector field to be parallel with respect to the torsion connection. In particular, this

vector field is an infinitesimal automorphism of the ACYT structure. In the case of closed torsion, the torsion connection is Ricci-flat if and only if either the norm of the torsion or the Riemannian scalar curvature is constant. On a compact ACYT 6-manifold the curvature of the torsion connection is symmetric on exchange of the two pairs (the first and the second) and has vanishing Ricci tensor if and only if it satisfies the Riemannian first Bianchi identity i.e. it is Kaehler-like.

Ljudmila Kamenova *Entire curves on holomorphic symplectic varieties*

Any holomorphic symplectic manifold contains entire curves as shown by Verbitsky using ergodicity, i.e., holomorphic symplectic manifolds are non-hyperbolic. More generally, together with S.Lu and Verbitsky (and later, with C.Lehn) we have established the Kobayashi conjectures in cases of Lagrangian fibrations. In this talk we shall explore generalizations of these results to primitive symplectic varieties. Together with C.Lehn we prove that if a primitive symplectic variety with second Betti number $b_2 \geq 5$ satisfies the rational SYZ conjecture, then it is hon-hyperbolic, and if $b_2 \geq 7$ then the Kobayashi pseudometric vanishes identically. In particular, this applies to all known examples of holomorphic symplectic manifolds. For Lagrangian fibrations with no multiple fibers in codimension one, we also have holomorphic dominability results with S.Lu, that imply the existence of a Zariski dense entire curve on a holomorphic symplectic manifold admitting such a Lagrangian fibration.

Ernesto Lupercio *Orbifold Chen-Ruan Cohomology and the Number 12*

In this talk, I will present an instance of the famous number 12 in orbifold cohomology and provide insights into its significance and implications.

Jihun Park *Simply-connected positive Sasakian 5-manifolds and log del Pezzo surfaces*

Sasakian geometry is a vibrant field at the intersection of differential geometry, topology, complex geometry, and algebraic geometry, with applications ranging from theoretical physics to geometric analysis. In this talk, we explore closed simply connected 5-manifolds capable of hosting positive Sasakian structures. We present a conjectural comprehensive list of such manifolds.

Tony Pantev TBA

Alexander Petkov *Li–Yau sub-gradient estimates and Perelman-type entropy formulas for the heat equation in quaternionic contact geometry*

We are going to present in this talk two sub-gradient estimates for the quaternionic contact (qc) heat equation on a compact qc manifold of dimension $4n + 3$, provided some positivity conditions are satisfied. These are qc versions of the prominent Li–Yau gradient estimate in Riemannian geometry. Another goal of the talk is to demonstrate two Perelman-type entropy formulas for the qc heat equation on a compact qc-Einstein manifold of dimension $4n + 3$ with non-negative qc scalar curvature (e.g. compact 3-Sasakian manifold), as well as an integral sub-gradient estimate for the positive solutions of the qc heat equation.

Vivek Shende TBA

Artan Sheshmani *Shifted symplectic structures on derived Quot Schemes, Degenerations and Categorification of DT invariants*

We discuss construction of a derived Lagrangian intersection theory of closely related moduli spaces of perfect complexes, with support on divisors on compact Calabi–Yau threefolds. Our goal is to compute deformation invariants associated to a fixed linear system of divisors in CY3. We degenerate the CY3 into a normal-crossing singular variety composed of Fano threefolds meeting along their anti-canonical divisor. The derived Lagrangian intersection of the corresponding "Fano moduli spaces" provides one with categorification of DT invariants over the special fiber (of degenrating family). We then elaborate on existence of a "Getzler–Gauss–Manin" connection on the Z/2-graded periodic cyclic homology associated to these categorified DT invariants. The latter provides the deformation invariance property, needed to relate the categorical DT invariants of the special fiber to the one over the generic fiber. If there is time, we also show how in terms of "non-derived" virtual interscetion theory these DT invariants relate to count of D4-D2-D0 branes which are expected to have modularity property by S-duality conjecture. This talk is based on joint work with Vladimir Baranovsky, Ludmil Katzarkov and Maxim Kontsevich.

Misha Shkolnikov *Motivic decomposition of generalized toric surfaces*

A classical compact symplectic toric surface, via the moment map, may be seen as a rational polygon on the plane with an integral affine structure. From this perspective, we may think of a generalized symplectic toric surface as represented by a general convex domain. The tropical caustic curves of these domains provide a natural geometric realization of such complex surfaces in the category of pro-orbifolds. Notably, the connected components in the complement of a caustic are again convex, and thus the notion of a secondary caustic makes sense for them. By iterating this construction for apeirogons that arise in the process, one gets a canonical decomposition of the domain into rational polygons, corresponding to classical toric surfaces. In my talk, I will review fundamental concepts of tropical optics

and address the question: which surfaces can appear in the iterated caustic decomposition?

Based on joint projects with Grigory Mikhalkin, Ernesto Lupercio, Nikita Kalinin, and Kevin Calderon.

Jason Starr *Complex Fano manifolds in the (very) large and holomorphic spheres.*

Deformations "in the large" of compact Kaehler manifolds extend both to "symplectic deformations" and "mixed characteristic" deformations. As Serre proved, even the fundamental group of Kaehler manifolds may change under mixed characteristic deformations, although the (profinite) etale homotopy type is preserved. Similarly, some holomorphic features are preserved by symplectic deformations. In this lecture, I review several features of complex Fano manifolds (and slight generalizations) that are preserved by both kinds of deformations, mostly related to holomorphic spheres / rational curves and Mori theory. The key idea is to use symplectic invariants, e.g., Gromov–Witten invariants, to control behavior when passing through positive characteristic.