



**DRP2025**  
10 – 14 March 2025



Monday		Tuesday		Wednesday		Thursday		Friday	
9:45 – 10:00	opening								
10:00 – 11:00	Browning	10:00 – 11:00	Newton	10:00 – 11:00	Pagano	10:00 – 11:00	Skorobogatov	10:00 – 11:00	Schindler
	coffee break		coffee break		coffee break		coffee break		coffee break
11:30 – 12:30	Tschinkel	11:30 – 12:30	Loughran	11:30 – 12:30	Sofos	11:30 – 12:30	Pieropan	11:30 – 12:30	Colliot-Thélène
	break		break		break		break		
12:35 – 13:05	Wang	12:35 – 13:05	Santens	12:35 – 13:05	Wilson	12:35 – 13:05	Darda		end of conference
	lunch break		lunch break				lunch break		
15:00 – 16:00	Derenthal	15:00 – 16:00	Javan Peykar			15:00 – 15:30	Gajović		
			coffee break			15:30 – 16:00	Uhlemann		
		16:30 – 17:00	Streeter				coffee break		
		17:00 – 17:30	Bartsch			16:30 – 17:00	Rome		
			break		free afternoon	17:00 – 17:30	Demeio		
17:00	Yuri Tschinkel's official lecture at BAS & reception	17:40 – 18:10	Sankar						
		18:15	Vlad's special reserve rakia tasting			19:30	conference dinner		

The conference is part of Horizon Europe MSCA Postdoctoral Fellowship project “Generalised Integrality and Applications to Number Theory” with PI: Vladimir Mitankin, funded by the European Union, Grant agreement 101151205 – GIANT.

The event is also funded by Simons Foundation and the Ministry of Education and Science of Bulgaria through the Scientific Program PIKOM.



## **Tim Browning**

*Pairs of commuting matrices*

I'll discuss commuting varieties and a new upper bound for the number of pairs of commuting  $n \times n$  matrices with integer entries and height at most  $T$ , as  $T \rightarrow \infty$ . Our approach uses Fourier analysis and mod  $p$  information, together with a result about the flatness of the commutator Lie bracket, which we also solve. This is joint work with Will Sawin and Victor Wang.

## **Yuri Tschinkel**

*Equivariant unirationality (joint with Cheltsov, Kresch, Zhang)*

I will discuss new ideas and constructions in equivariant birational geometry, with special regard for cubic threefolds.

## **Victor Wang**

*Some examples of symmetry*

I will discuss some arithmetic problems, about Diophantine equations and/or  $L$ -functions, where symmetry is either built-in or otherwise potentially relevant.

## **Ulrich Derenthal**

*Integral points on a family of spherical Fano threefolds*

The asymptotic behavior of the number of rational points of bounded anticanonical height on Fano varieties over number fields is predicted by Manin's conjecture. For integral points of bounded height, one might expect a similar behavior within a framework developed by Chambert-Loir and Tschinkel. In joint work in progress with Florian Wilsch, we prove such asymptotic formulas for integral points on a family of singular spherical Fano threefolds, with respect to arbitrary polarizations.

## **Rachel Newton**

### *Counting $S_4$ and $S_5$ extensions satisfying the Hasse norm principle*

Let  $L/K$  be an extension of number fields. The norm map  $N_{L/K} : L^* \rightarrow K^*$  extends to a norm map from the ideles of  $L$  to those of  $K$ . The Hasse norm principle is said to hold for  $L/K$  if, for elements of  $K^*$ , being in the image of the idelic norm map is equivalent to being the norm of an element of  $L^*$ . The frequency of failure of the Hasse norm principle in families of abelian extensions is fairly well understood, thanks to previous work of Christopher Frei, Daniel Loughran and myself, as well as more recent work of Peter Koymans and Nick Rome. In this talk, I will focus on the non-abelian setting and discuss joint work with Ila Varma on the statistics of the Hasse norm principle in field extensions with normal closure having Galois group  $S_4$  or  $S_5$ .

## **Daniel Loughran**

### *Distribution of class group via stacks*

In this talk I will explain how the Cohen-Lenstra heuristics on distributions of class groups can be viewed as a version of Peyre's equidistribution conjectures for rational points on the stack  $BG$ . This is joint work with Tim Santens and Ross Paterson.

## **Tim Santens**

### *Local solubility of generalised Fermat equations*

Let  $n > 1$ . I will discuss how many of the Fermat curves  $ax^n + by^n + cz^n = 0$  are everywhere locally soluble as  $a, b, c$  varies. This is joint work with Peter Koymans, Ross Paterson and Alec Shute.

## **Ariyan Javanpeykar**

*The weakly special conjecture contradicts Orbifold Mordell (and hence abc)*

Lang conjectured that varieties of general type over a number field have very few rational points. In 2000, guided by Lang's conjecture and in search of a converse statement, Abramovich, Colliot-Thélène, Harris, and Tschinkel formulated the "Weakly Special Conjecture": every weakly special variety over a number field has a potentially dense set of rational points. In this talk I will explain how this conjecture contradicts the *abc* conjecture, and more precisely Campana's "Orbifold Mordell" conjecture. Indeed, starting from an Enriques surface over  $Q(t)$  constructed by Lafon, we give the first examples of smooth projective weakly special threefolds which fiber over the projective line in Enriques surfaces with nowhere reduced, but non-divisible, fibers. The existence of these threefolds shows that the Weakly Special Conjecture contradicts the *abc* conjecture, but also shows that Enriques surfaces and K3 surfaces can have non-divisible but nowhere reduced degenerations, thereby answering a question raised by Campana in 2005. This is joint work with Finn Bartsch, Frederic Campana, and Olivier Wittenberg.

## **Sam Streeter**

*Semi-integral points on toric varieties*

We review recent progress on conjectures on the distribution of rational points in the context of semi-integral (Campana and Darmon) points. In particular, we establish a semi-integral analogue of Manin's conjecture for toric varieties in the spirit of conjectures of Pieropan–Smeets–Tanimoto–Várilly-Alvarado and Chow–Loughran–Takloo-Bighash–Tanimoto. This is joint work with Alec Shute.

## **Finn Bartsch**

*Symmetric products and puncturing Campana-special varieties*

In 2001, Hassett and Tschinkel posed the following "puncturing problem": If  $X$  is a projective variety with at most canonical singularities such that no finite étale cover of  $X$  dominates a variety of general type and  $Z$  is a closed subset of  $X$  of codimension at least 2, does it follow that no finite étale cover of  $X \setminus Z$  dominates a variety of log-general type? Following the philosophy that maps to varieties of general type should be the main obstruction to density of rational points, they also suggested the following "arithmetic puncturing problem": With  $X$  and  $Z$  as above, if the rational points on  $X$  are potentially dense, are the integral points on  $X \setminus Z$  potentially dense? In this talk, I will explain how symmetric powers of products of curves provide counterexamples to both of these puncturing problems. On the other hand, conjectures of Campana suggest that the arithmetic puncturing problem has a positive answer if we additionally assume  $X$  to be smooth. This is joint work with Ariyan Javanpeykar and Aaron Levin.

## **Soumya Sankar**

### *On the rationality of conic bundle threefolds*

One of the major programs in algebraic geometry seeks to understand when a given variety is rational. From this perspective, conic bundles are a structurally simple, but geometrically rich class of varieties. One may ask, what does one need to know about a conic bundle in order to determine if it is rational? The answer to this question, at least in low dimensions, is well understood over algebraically closed fields. In this talk, I will discuss what is known about conic bundle threefolds over non-algebraically closed fields: how the usual refinement of the algebraically closed case is not enough, and what additional information one might need to answer the question. This talk is based on joint work with Sarah Frei, Lena Ji, Bianca Viray and Isabel Vogt.

## **Carlo Pagano**

### *Hilbert 10 via additive combinatorics*

In 1970 Matiyasevich, building on earlier work of Davis–Putnam–Robinson, proved that every enumerable subset of  $\mathbb{Z}$  is Diophantine, thus showing that Hilbert’s 10th problem is undecidable for  $\mathbb{Z}$ . The problem of extending this result to the ring of integers of number fields (and more generally to finitely generated infinite rings) has attracted significant attention and, thanks to the efforts of many mathematicians, the task has been reduced to the problem of constructing, for certain quadratic extensions of number fields  $L/K$ , an elliptic curve  $E/K$  with  $\text{rk}(E(L)) = \text{rk}(E(K)) > 0$ . In this talk I will explain joint work with Peter Koymans, where we use Green–Tao to construct the desired elliptic curves, settling Hilbert 10 for every finitely generated infinite ring.

## **Efthymios Sofos**

### *Ranks of elliptic fibrations*

In the 1950s, Erdős developed a method to estimate the average of the divisor function over the values of an integer polynomial. Nair and Tenenbaum later extended this to a substantially general class of arithmetic functions. In 1993 Heath-Brown used character sums to study the average size of the 2-Selmer group in  $ty^2 = x^3 - x$ . Combining these approaches, we prove that all exponential moments of the rank of  $P(t)y^2 = x^3 - x$  are bounded, where  $P$  is an arbitrary polynomial. This is joint work with Peter Koymans and Carlo Pagano.

## **Cameron Wilson**

### *Local solubility of quadratic forms over a biprojective base*

I will give a brief overview of the problem of local solubility for algebraic varieties in families before discussing some of my recent work on this problem regarding families of quadratic forms parameterised by  $\mathbb{P}^1 \times \mathbb{P}^1$ .

## Alexei Skorobogatov

### *Markoff surfaces and birational properties of word varieties*

Markoff surfaces naturally appear in the classification of pairs of matrices in  $SL(2)$  up to simultaneous similarity. I will explain how the subvariety of  $SL(2) \times SL(2)$  given by  $w(X, Y) = A$ , where  $w$  is a word in two letters and  $A$  is a fixed matrix, is related to a conic bundle over a surface in the 3-dimensional affine space ramified in its intersection with the Markoff surface. In the case of the commutator word, this variety can be non-rational, answering a question from 1996. This is joint work with Bandman and Kunyavskii.

## Marta Pieropan

### *Lifting rational points along log smooth morphisms*

Determining the image of the set of rational points under a morphism of varieties is a very natural and difficult question. The case where the morphism is geometrically surjective has been studied extensively. From Campana's theory of orbifolds, it follows that over a number field, the image of the set of rational points is contained in the set of Campana points for the orbifold base of the morphism. This first approximation of the image of the set of rational points is refined by Abramovich's theory of firmaments. This talk presents joint work with Herr, Mehidi and Poiret that addresses the same questions in the setting of log schemes and provides a proof of a claim by Abramovich about lifting firm points under toroidal morphisms.

## Ratko Darda

### *Arithmetic statistics and wild stacks*

Recent works have extended the (Batyrev)–Manin conjecture on the number of rational points on varieties to Deligne–Mumford stacks. As a special case, the general conjecture recovers Malle's conjecture on the number of Galois extensions of bounded discriminant. The approach for number fields works for tame stacks, that is, when the orders of automorphism groups of points are coprime to the characteristic of the base field.

In this talk, we look at what happens in the wild (non-tame) setting. We will primarily focus on the 0-dimensional case, which contains Malle's conjecture. The talk is based on joint work with Takehiko Yasuda.

## Stevan Gajović

*Images of certain  $p$ -adic polynomials, their ratio sets, and a conjecture in additive combinatorics*

For a given polynomial  $f$  in  $\mathbb{Z}_p[x]$ , we investigate if we can solve the equation  $f(x)/f(y) = s$  for all  $s$  in  $\mathbb{Q}_p$ , where  $x$  and  $y$  are in  $\mathbb{Z}_p$ . Miska, Murru, and Sanna proved that the answer is yes if  $f$  has a simple root or, more generally, if  $f$  has two roots with coprime multiplicities. Let  $q > 1$  be an integer. We consider polynomials which are a product of the  $q$ th power of a polynomial and a product of irreducible polynomials whose degrees are divisible by  $q$ . We give a criterion for when the answer to the starting question is no and show that this criterion is sharp by providing examples with the minimal number of such factors when the answer is yes; this is related to a conjecture in additive combinatorics. We give a criterion for polynomials of small degree. This is joint work with Deepa Antony and Rupam Barman.

## Justin Uhlemann

*Local-global principles for Campana points on Markoff surfaces*

In 2017, Ghosh and Sarnak investigated the set of integral points on certain families of affine cubic surfaces, establishing explicit sub families that give rise to integral Hasse failures. Follow up work of Loughran–Mitankin, and Colliot-Thélène–Wei–Xu studied the quantity of these Hasse failures explained by an integral version of Brauer–Manin obstruction. In this talk, we use the notion of Campana orbifold pairs to study local-global principles for Campana points, which unify the notion of integral and rational points. We provide an almost sharp lower bound for the number of orbifold pairs that satisfy a variant of the Hasse principle for Campana points. This is based on joint work with Vlad Mitankin.

## Nick Rome

*Quadratic points on surfaces*

I will discuss the Manin–Peyre conjecture for the symmetric squares of Fano surfaces. We give a framework to prove results of this kind, including results on summing Euler products over quadratic extensions. Moreover, I'll discuss a family of examples where we are able to prove the conjecture via counting quadratic points on quadric surfaces with uniform dependence on the underlying field.

**Julian Demeio***Hilbert Property for a family of del Pezzos of degree 1*

Abundance of rational points on del Pezzos of degree 1 is a notoriously hard problem as even the unirationality of these geometrically rational surfaces is still an open problem. Recently Nijgh proved Zariski-density on a certain subfamily of these generalizing an earlier result of Desjardins and Winter. We show that in his family the rational points are also not thin. This is work in progress with Sam Streeter and Rosa Winter.

## **Damaris Schindler**

### *Quantitative weak approximation and quantitative strong approximation for certain quadratic forms*

In this talk we discuss recent results on optimal quantitative weak approximation for certain projective quadrics over the rational numbers as well as quantitative results on strong approximation for quaternary quadratic forms over the integers. We will also discuss results on the growth of integral points on the three-dimensional punctured affine cone and strong approximation with Brauer–Manin obstruction for this quasi-affine variety. This is joint work with Zhizhong Huang and Alec Shute.

## **Jean-Louis Colliot-Thélène**

### *On the rationality of real quadric bundles*

A real geometrically (smooth, projective) rational surface is rational over the reals if and only if its set of real points is nonempty and connected. This need not hold for higher dimensional geometrically rational real varieties. Results for specific classes of threefolds will be recalled. In joint work with Alena Pirutka, we examine the rationality problem for very simple looking threefolds with a pencil of quadrics and produce an unramified cohomology invariant whose vanishing characterizes decomposition of the diagonal for the threefold. In many cases we prove that it vanishes.